APPLIED ECONOMICS
https://doi.org/10.1080/00036846.2019.1682116

Avoiding Mis-estimation of the CES Function: Unit Matters

Haoqi Qian[a,b] and Libo Wu[c,d]

[a]Institute for Global Public Policy, Fudan University, Shanghai, China; [b]LSE-Fudan Research Centre for Global Public Policy, Fudan University, Shanghai, China; [c]School of Economics, Fudan University, Shanghai, China; [d]School of Data Science, Fudan University, Shanghai, China

ABSTRACT
We demonstrate that unit errors of measurement will lead to significant biases in estimating the constant elasticity of substitution (CES) function. Monte-Carlo simulations show that estimation results tend to reach Cobb–Douglas (CD) functions or extreme values if units of input variables are incorrectly used. To avoid this problem, we suggest adding an overall efficiency parameter and a unit correction parameter which is similar to biased technological change parameter when estimating CES functions. Any unit error of measurement can be captured by these two parameters while allowing researchers to get unbiased estimation results of other parameters.

KEYWORDS
Constant elasticity of substitution; CES function; unit error; non-linear least squares

JEL CLASSIFICATION
C13; C20; C15; C51

I. Introduction

Direct estimation of nonlinear constant elasticity of substitution (CES) function is gaining increasing popularities in recent years. From theoretical point of view, direct estimation can avoid price distortion problems (Papageorgiou, Saam and Schulte 2017) and facilitate to identify biased technological changes (Klump, McAdam, and Willman 2007a, 2007b). Due to the nonlinearity of CES function, key parameters in CES function are no longer 'deep' parameters in that they depend on units of measurement of inputs (Temple 2012). However, effects of unit errors on estimation results have been seldom discussed in existing literature, and currently there exists no state-of-the-art approaches to solve this problem.

In this article, we investigate the unit error problem in estimating CES functions in Section 2 and 3. Section 4 provides the solution to overcome this problem and discussions of potential advantages of the solution. Section 5 is the conclusion.

II. Basic model

The basic CES function (Arrow et al. 1961) allowing biased technological changes (Antràs 2004) is written as:

$$Y_t = \left[\alpha\left(A_t X_{1,t}\right)^{-\rho} + (1-\alpha)\left(B_t X_{2,t}\right)^{-\rho}\right]^{-\lambda/\rho} \quad (1)$$

where subscript t denotes time. Y is output variable, X_1 and X_2 are input variables. A_t and B_t are biased technological change parameters of X_1 and X_2. α is the distribution parameter, λ is the scale parameter and ρ is the substitution parameter. The elasticity of substitution between two inputs is defined as $\sigma = 1/(1+\rho)$.

Instead of estimating formula (1) directly, incorporating optimization conditions into formula (1) can make the estimation results more consistent to real data. The objective function faced by the representative firm in period t is:

$$\begin{array}{ll} \max\limits_{X_{1,t},X_{2,t}} & \left[\alpha\left(A_t X_{1,t}\right)^{-\rho} + (1-\alpha)\left(B_t X_{2,t}\right)^{-\rho}\right]^{-\lambda/\rho} \\ s.t. & p_{1,t}X_{1,t} + p_{2,t}X_{2,t} = M_t \end{array} \quad (2)$$

where $p_{1,t}$ and $p_{2,t}$ are prices of two inputs, M_t is the cost budget. Combining two first-order conditions $\partial\ell/\partial X_1 = 0$ and $\partial\ell/\partial X_2 = 0$, we solve for α as:

$$\alpha = \frac{\pi_t\left(A_t X_{1,t}\right)^{\rho}}{\pi_t\left(A_t X_{1,t}\right)^{\rho} + (1-\pi_t)\left(B_t X_{2,t}\right)^{\rho}} \quad (3)$$

where $\pi_t = p_{1,t}X_{1,t}/\left(p_{1,t}X_{1,t} + p_{2,t}X_{2,t}\right)$. Replacing formula (3) into formula (1) gives:

$$Y_t = \left[\pi_t\left(A_t X_{1,t}\right)^{\rho} + (1-\pi_t)\left(B_t X_{2,t}\right)^{\rho}\right]^{\lambda/\rho} \quad (4)$$

CONTACT Haoqi Qian qianhaoqi@fudan.edu.cn Institute for Global Public Policy, Fudan University, Handan Road 220#, Shanghai, China; LSE-Fudan Research Centre for Global Public Policy, Fudan University, Handan Road 220#, Shanghai, China

Table 1. Settings of Monte-Carlo simulations.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
ρ	−0.6	−0.6	−0.6	1.0	1.0	1.0
λ	1.05	1.05	1.05	0.98	0.98	0.98
μ_1	0.002	0.002	n.a.	0.003	0.003	n.a.
μ_2	0.004	n.a.	0.004	0.001	n.a.	0.001
π		$\pi_t = 0.15 + 0.5\pi_{t-1} + N(0, 0.01^2)$			$\pi_t = 0.35 + 0.5\pi_{t-1} + N(0, 0.01^2)$	
X_1		$N(1, 0.1^2)$			$N(1, 0.1^2)$	
X_2		$N(1, 0.1^2)$			$N(1, 0.1^2)$	
Error		$N(0, 0.01^2)$			$N(0, 0.01^2)$	
Draw		1000			1000	
T		200			200	

$N(\varphi, \sigma^2)$ is the normal distribution with mean φ and standard deviation σ.
'n.a.' means no biased technological changes.

In the new representation of CES function described in formula (4) the distribution parameter is dropped and is replaced by cost share of input X_1 in period t. Thus, the first-order conditions are embedded in the new CES function and it can be easily shown that estimating formula (4) is equivalent to the supply side system approach first proposed by Klump, McAdam and Willman (2007a).

III. Effects of unit errors on estimation results

In formula (4) four parameters to be estimated actually depend on the unit of measurements of input variables. To see this, we assume formula (4) is the true underlying CES function, but $q_1 X_{1,t}$ and $q_2 X_{2,t}$ are instead used to estimate it with $q_1, q_2 \in (0, +\infty)$ and $q_1/q_2 = q$. Thus, q is an exogenous parameter which measures the degree of relative unit error. Biased technological changes are commonly assumed to grow at a constant rate, so $A_t = A_0 e^{\mu_1 t}$ and $B_t = B_0 e^{\mu_2 t}$. Here, we first set $A_0 = B_0 = 1$ for simplicity and discuss them later. Then the CES function to be estimated becomes:

$$Y = \left[\pi_t \left(q_1 e^{\mu_1 t} X_{1,t}\right)^\rho + (1 - \pi_t)\left(q_2 e^{\mu_2 t} X_{2,t}\right)^\rho\right]^{\lambda/\rho}$$
$$= \eta\left[\pi_t \left(q e^{\mu_1 t} X_{1,t}\right)^\rho + (1 - \pi_t)\left(e^{\mu_2 t} X_{2,t}\right)^\rho\right]^{\lambda/\rho} \quad (5)$$

where $\eta = q_2^\lambda$ which means unit error of measurement of X_2 is absorbed by η. Then, the NLS estimation is to minimize the sum of squared residuals (SSR) as follows:

$$SSR = \sum_{t=1}^{T} \left\{ Y_t - \eta\left[\pi_t \left(q e^{\mu_1 t} X_{1,t}\right)^\rho + (1 - \pi_t)\left(e^{\mu_2 t} X_{2,t}\right)^\rho\right]^{\lambda/\rho} \right\}^2 \quad (6)$$

where subscript t denotes observation in period t and total period is T. It is clearly seen that the estimation results are highly dependent on q. To see this, we conduct six Monte-Carlo simulations in order to investigate the effects of different unit of measurements. In all six cases, we assume X_2 is correctly measured and X_1 is incorrectly measured from 100 times smaller to 100 times larger than true value, thus $0.01 \le q \le 100$. The true data generating process (DGP) follows formula (4) with an additive error term, parameters and data used in six simulations are shown in Table 1.

The NLS estimating procedure follows Henningsen and Henningsen (2012), so rounding errors that will occur when ρ approaches zero can be avoided. Besides, the Trust Region algorithm are used to avoid the drawbacks of Levenberg–Marquardt (LM) algorithm used in Henningsen, Henningsen and van der Werf (2018) that it may fail to convergent when starting values are too far from the global optimum. Simulation results are illustrated in Figure 1.

From Figure 1, we can see that if X_1 is correctly measured, i.e. $q = 1$, then expected value of the substitution parameter $\hat{\rho}$ will be close to its true value. However, if degree of unit error increases, i.e. $q \to 0$ or $q \to +\infty$, expected values of $\hat{\rho}$ will approach to zero or reach extreme values. Meanwhile, the corresponding goodness of fit, R^2, also decreases dramatically. As a result, if researchers use wrong unit of measurements of inputs, they will tend to either obtain Cobb–Douglas (CD) functions or unreasonable values. This will raise great concerns that empirical works in estimating CES functions may give biased estimation results if unit of measurements are not carefully treated. This result is similar to the statements in Antràs (2004) that if biased technological changes are omitted in CES function, then estimation results will necessarily be a CD function.

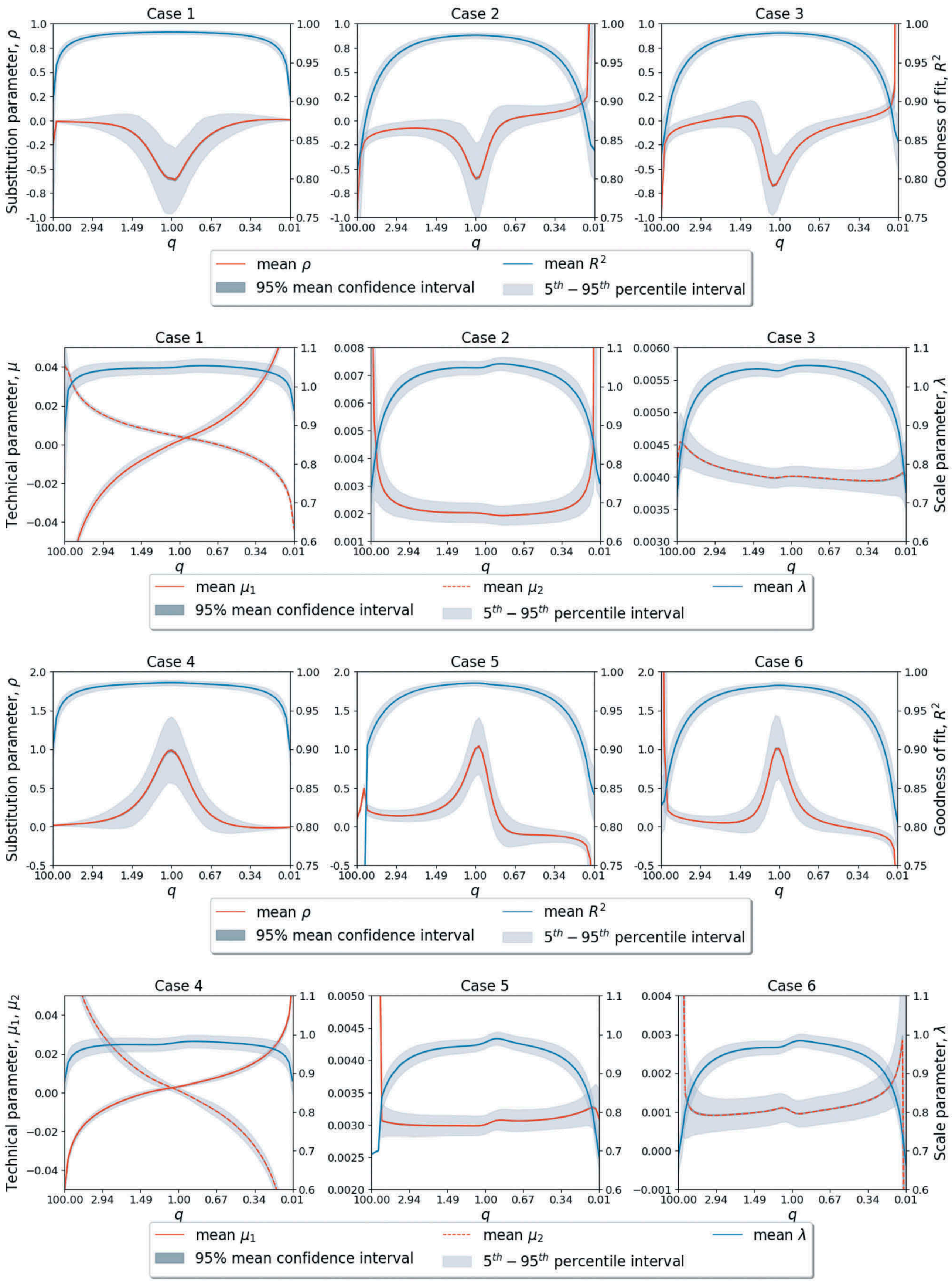

Figure 1. Estimated parameters with different unit of measurements.

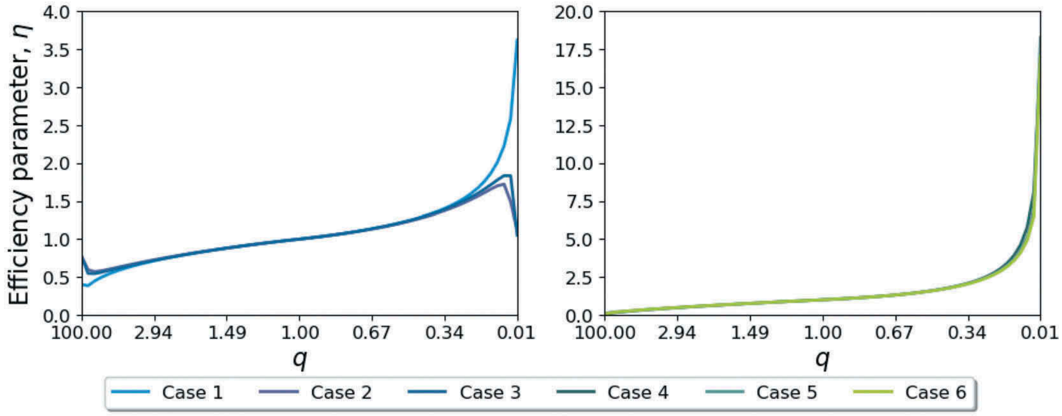

Figure 2. Estimated efficiency parameters with different unit of measurements.

For those technical parameters μ and scale parameters λ, estimation results are also greatly affected by unit errors whenever we assume both inputs or only one of them have biased technological changes. As a result, all remaining variations are captured by the efficiency parameter η. In Figure 2, we can see that expected values of $\hat{\eta}$ deviate in a wide range as q deviates from unity.

IV. Discussions

Modified specifications of CES function

Now come back to formula (5) if we add back A_0 and B_0 then formula (5) becomes:

$$\begin{aligned} Y &= \left[\pi_t\left(q_1 A_0 e^{\mu_1 t} X_{1,t}\right)^\rho + (1-\pi_t)\left(q_2 B_0 e^{\mu_2 t} X_{2,t}\right)^\rho\right]^{\lambda/\rho} \\ &= \gamma\left[\pi_t\left(\tau e^{\mu_1 t} X_{1,t}\right)^\rho + (1-\pi_t)\left(e^{\mu_2 t} X_{2,t}\right)^\rho\right]^{\lambda/\rho} \qquad (7) \end{aligned}$$

where $\gamma = (q_2 B_0)^\lambda$ and $\tau = qA_0/B_0$. Constant components of biased technological changes can be absorbed with γ and τ, so it is no longer needed to set them explicitly in the CES functions. By adding an overall efficiency parameter γ and a unit correction parameter τ, unit errors can be well captured by these two parameters whilst keep other parameters unchanged. Table 2 summarizes specifications of CES functions under different situations. These specifications are to some extent explicitly or inexplicitly applied in some existing literature (Antràs 2004; van der Werf, 2008). For the empirical literature which does not apply these specifications, it is more likely to find estimation results with extreme values (Prywes 1986; Kemfert 1998; Koesler and Schymura 2015).

Challenges in estimating CES functions

Unit errors may sometimes be quite large. For example, if one changes the unit from thousand to million of one input while keep another input's unit unchanged, then τ is expected to become 1000 times larger. New challenges may be raised in applying NLS method in that the start values will be too far from the global optimum which leads to failure in optimization procedure. This problem is sometimes more severe than expected in that even Trust Region algorithm is not able to find the global optimum. To overcome this problem, a thorough grid search for η and τ is needed to find the most potential start values within a wide range of parameter spaces. Although a thorough grid search will be computation intensive and time

Table 2. Parameters and data used in Monte-Carlo simulations.

Biased technological change	Specification of CES function	Parameters to be estimated
X_1, X_2	$Y = \gamma\left[\pi_t\left(\tau e^{\mu_1 t} X_{1,t}\right)^\rho + (1-\pi_t)\left(e^{\mu_2 t} X_{2,t}\right)^\rho\right]^{\lambda/\rho}$	$\gamma\ \lambda\ \mu_1\ \mu_2\ \tau\ \rho$
X_1	$Y = \gamma\left[\pi_t\left(\tau e^{\mu t} X_{1,t}\right)^\rho + (1-\pi_t) X_{2,t}^\rho\right]^{\lambda/\rho}$	$\gamma\ \lambda\ \mu\ \tau\ \rho$
X_2	$Y = \gamma\left[\pi_t\left(\tau X_{1,t}\right)^\rho + (1-\pi_t)\left(e^{\mu t} X_{2,t}\right)^\rho\right]^{\lambda/\rho}$	$\gamma\ \lambda\ \mu\ \tau\ \rho$
None	$Y = \gamma\left[\pi_t\left(\tau X_{1,t}\right)^\rho + (1-\pi_t) X_{2,t}^\rho\right]^{\lambda/\rho}$	$\gamma\ \lambda\ \tau\ \rho$

Table 3. Monte-Carlo simulation with parameters in case 1.

	R^2	τ	γ	ρ	μ_1	μ_2	λ
True value	-	$1/q_1$	**1.000**	**−0.600**	**0.002**	**0.004**	**1.050**
$q_1 = 100, q_2 = 1$	**0.989** (0.002)	**0.010** (0.001)	**1.001** (0.024)	**−0.590** (0.217)	**0.002** (0.002)	**0.004** (0.001)	**1.050** (0.009)
$q_1 = 10, q_2 = 1$	**0.989** (0.002)	**0.100** (0.007)	**1.001** (0.023)	**−0.597** (0.224)	**0.002** (0.002)	**0.004** (0.001)	**1.050** (0.009)
$q_1 = 1, q_2 = 1$	**0.989** (0.002)	**1.001** (0.074)	**1.001** (0.024)	**−0.589** (0.216)	**0.002** (0.002)	**0.004** (0.001)	**1.050** (0.009)
$q_1 = 0.1, q_2 = 1$	**0.989** (0.002)	**10.007** (0.744)	**1.001** (0.024)	**−0.589** (0.218)	**0.002** (0.002)	**0.004** (0.001)	**1.050** (0.009)
$q_1 = 0.01, q_2 = 1$	**0.989** (0.002)	**100.179** (7.266)	**1.001** (0.023)	**−0.591** (0.217)	**0.002** (0.002)	**0.004** (0.001)	**1.050** (0.009)
True value	-	q_2	$q_2^{-\lambda}$	**−0.600**	**0.002**	**0.004**	**1.050**
$q_1 = 1, q_2 = 100$	**0.988** (0.012)	**99.992** (7.116)	**0.008** (0.000)	**−0.610** (0.203)	**0.002** (0.002)	**0.004** (0.001)	**1.050** (0.009)
$q_1 = 1, q_2 = 10$	**0.989** (0.005)	**9.996** (0.717)	**0.089** (0.003)	**−0.600** (0.231)	**0.002** (0.002)	**0.004** (0.001)	**1.050** (0.009)
$q_1 = 1, q_2 = 1$	**0.989** (0.002)	**0.999** (0.073)	**1.002** (0.023)	**−0.609** (0.203)	**0.002** (0.002)	**0.004** (0.001)	**1.050** (0.009)
$q_1 = 1, q_2 = 0.1$	**0.979** (0.027)	**0.101** (0.006)	**11.236** (0.386)	**−0.608** (0.215)	**0.002** (0.002)	**0.004** (0.001)	**1.050** (0.009)
$q_1 = 1, q_2 = 0.01$	**0.979** (0.027)	**0.010** (0.001)	**126.189** (6.569)	**−0.610** (0.203)	**0.002** (0.002)	**0.004** (0.001)	**1.050** (0.009)

Standard deviations are provided in parentheses.

Table 4. Monte-Carlo simulation with parameters in case 4.

	R^2	τ	γ	ρ	μ_1	μ_2	λ
True value	-	$1/q_1$	**1.000**	**1.000**	**0.003**	**0.001**	**0.980**
$q_1 = 100, q_2 = 1$	**0.962** (0.075)	**0.010** (0.000)	**1.000** (0.036)	**0.998** (0.253)	**0.003** (0.001)	**0.001** (0.002)	**0.980**(0.010)
$q_1 = 10, q_2 = 1$	**0.962** (0.075)	**0.101** (0.004)	**1.000** (0.036)	**0.998** (0.254)	**0.003** (0.001)	**0.001** (0.002)	**0.980** (0.010)
$q_1 = 1, q_2 = 1$	**0.986** (0.002)	**1.002** (0.052)	**1.000** (0.036)	**0.998** (0.254)	**0.003** (0.001)	**0.001** (0.002)	**0.980** (0.010)
$q_1 = 0.1, q_2 = 1$	**0.986** (0.002)	**10.017**(0.519)	**1.000** (0.036)	**0.998** (0.254)	**0.003** (0.001)	**0.001** (0.002)	**0.980** (0.010)
$q_1 = 0.01, q_2 = 1$	**0.986** (0.002)	**99.746** (4.836)	**1.003** (0.033)	**1.000** (0.253)	**0.003** (0.001)	**0.001** (0.002)	**0.980** (0.010)
True value	-	q_2	$q_2^{-\lambda}$	**1.000**	**0.003**	**0.001**	**0.980**
$q_1 = 1, q_2 = 100$	**0.964** (0.065)	**101.400** (4.348)	**0.011** (0.001)	**1.006** (0.241)	**0.003** (0.001)	**0.001** (0.002)	**0.981** (0.010)
$q_1 = 1, q_2 = 10$	**0.964** (0.065)	**10.142** (0.434)	**0.104** (0.004)	**1.006** (0.241)	**0.003** (0.001)	**0.001** (0.002)	**0.981** (0.010)
$q_1 = 1, q_2 = 1$	**0.986** (0.002)	**1.006** (0.053)	**0.998** (0.036)	**1.006** (0.241)	**0.003** (0.001)	**0.001** (0.002)	**0.981** (0.010)
$q_1 = 1, q_2 = 0.1$	**0.964** (0.065)	**0.101** (0.004)	**9.543** (0.428)	**1.006** (0.241)	**0.003** (0.001)	**0.001** (0.002)	**0.981** (0.010)
$q_1 = 1, q_2 = 0.01$	**0.964** (0.065)	**0.010** (0.000)	**91.320** (5.586)	**1.009** (0.240)	**0.003** (0.001)	**0.001** (0.002)	**0.981** (0.010)

Standard deviations are provided in parentheses.

consuming, it is worthwhile to obtain reliable estimation results by doing so. We demonstrate the above statements by conducting Monte-Carlo simulations with settings in case 1 and case 4, the only exception is that an additional parameter τ is added in the estimation procedure. Simulation results are shown in Tables 3 and 4.

Discussion of normalization method

Among existing literature, Leon-Ledesma and Satchi (2011) and Cantore and Levine (2012) suggest to normalize data to the baseline point to overcome unit error problems. Normalized CES function is first formally proposed by Klump and Preissler (2000) which can be written as follows:

$$\frac{Y_t}{Y_0} = \left[\pi_0\left(\frac{A_tX_{1,t}}{X_{1,0}}\right)^{-\rho} + (1-\pi_0)\left(\frac{B_tX_{2,t}}{X_{2,0}}\right)^{-\rho}\right]^{-\lambda/\rho} \tag{8}$$

where subscript zero denotes benchmark value, and point $(Y_0, X_{1,0}, X_{2,0}, \pi_0)$ is called the normalization point. By viewing input variables as indices, we can make the CES function be invariant to changes of units of measurement (Klump, McAdam, and Willman 2012). However, Temple (2012) points out that the function surface of formula (8) is very sensitive to the choice of normalization point and the normalization method is not enough to make parameters 'deep'. In empirical studies, the normalization point may be chosen arbitrarily since the true normalization point is unknown to researchers. For example, normalization point is either chosen as the baseline year (Klump, McAdam, and Willman 2007b; León-Ledesma, McAdam, and Willman 2015) or as the average of sample data (Klump, McAdam, and Willman 2007a, 2012). Different choices of normalization point will lead to different estimation results which may reduce the reliabilities of results obtained by using normalization method.

Advantages of the new specifications in estimating nested CES function

Nested CES function is widely used in many economic fields such as climate change, international trade and so on. It is also the very basic functional form used in many large-scale simulation models. Consider the following modified two-level nested CES function:

$$Y_t = \gamma_1\left[\pi_{1,t}\left(\tau_1 A_t X_{1,t}\right)^{\rho_1} + \left(1-\pi_{1,t}\right)Z_t^{\rho_1}\right]^{\lambda/\rho_1}$$
$$Z_t = \left[\pi_{2,t}\left(\tau_2 B_t X_{2,t}\right)^{\rho_2} + \left(1-\pi_{2,t}\right)\left(C_t X_{3,t}\right)^{\rho_2}\right]^{1/\rho_2} \quad (9)$$

where A_t, B_t and C_t are biased technological change of X_1, X_2 and X_3. If units of X_1 and X_2 are incorrectly measured, then these errors will be captured by τ_2 and an omitted efficiency parameter γ_2 related with Z. Since γ_2 is omitted in formula (9) this means Z is also incorrectly measured. However, this error, together with unit error of X_1, can be captured by τ_1 and γ_1. As a result, whenever unit errors happen to any input variable in a nested CES function, these errors can be successfully captured by γ and τ that allow remaining substitution parameter ρ and scale parameter λ be deep parameters.

V. Conclusion

We have demonstrated that using incorrect units of measurement will lead to significant biases in estimating key parameters of CES functions. Researchers will obtain either CD functions or extreme estimation results if these errors are neglected in practice. To overcome this problem, we suggest specify the CES function by adding an overall efficiency parameter and a unit correction parameter to the CES function. By doing so, unit errors can be captured by these two new parameters through well conducted estimation procedure. When compared to normalized CES method, the new specification does not impose assumptions of benchmark points which may be chosen arbitrarily in empirical studies. Furthermore, the new specification also has advantages in estimating nested CES function since all unit errors can be absorbed by additional parameters.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported by the National Natural Science Foundation of China [Grant number 71703027]; The Key Program of National Philosophy and Social Science Foundation Grant [Grant number 15ZDB148].

ORCID

Haoqi Qian http://orcid.org/0000-0001-5156-4402

References

Antràs, P. 2004. "Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution." *Contributions to Macroeconomics* 4 (1): 1161.

Arrow, K. J., H. B. Chenery, B. S. Minhas, and R. M. Solow. 1961. "Capital-Labor Substitution and Economic Efficiency." *The Review of Economics and Statistics* 43 (3): 225–250.

Cantore, C., and P. Levine. 2012. "Getting Normalization Right: Dealing with "dimensional Constants" in Macroeconomics." *Journal of Economic Dynamics and Control* 36 (12): 1931–1949.

Henningsen, A., and G. Henningsen. 2012. "On Estimation of the CES Production function—Revisited." *Economics Letters* 115 (1): 67–69.

Henningsen, A., G. Henningsen, and E. van der Werf. 2018. "Capital-labour-energy Substitution in A Nested CES Framework: A Replication and Update of Kemfert (1998)." *Energy Economics*. doi:10.1016/j.eneco.2017.12.019.

Kemfert, C. 1998. "Estimated Substitution Elasticities of a Nested CES Production Function Approach for Germany." *Energy Economics* 20 (3): 249–264.

Klump, R., P. McAdam, and A. Willman. 2007a. "Factor Substitution and Factor-Augmenting Technical Progress in the United States: A Normalized Supply-Side System Approach." *The Review of Economics and Statistics* 89 (1): 183–192.

Klump, R., P. McAdam, and A. Willman. 2007b. "The Long-term sucCESs of the Neoclassical Growth Model." *Oxford Review of Economic Policy* 23 (1): 94–114.

Klump, R., P. McAdam, and A. Willman. 2012. "The Normalized CES Production Function: Theory and Empirics." *Journal of Economic Surveys* 26 (5): 769–799.

Klump, R., and H. Preissler. 2000. "CES Production Functions and Economic Growth." *Scandinavian Journal of Economics* 102 (1): 41–56.

Koesler, S., and M. Schymura. 2015. "Substitution Elasticities in a Constant Elasticity of Substitution Framework – Empirical Estimates Using Nonlinear Least Squares." *Economic Systems Research* 27 (1): 101–121.

León-Ledesma, M. A., P. McAdam, and A. Willman. 2015. "Production Technology Estimates and Balanced Growth." *Oxford Bulletin of Economics and Statistics* 77 (1): 40–65.

Leon-Ledesma, M. A., and M. Satchi. 2011. *The Choice of CES Production Techniques and Balanced Growth. Studies in*

Economics 1113. Department of Economics, University of Kent. https://www.kent.ac.uk/economics/documents/research/papers/2011/1113.pdf.

Papageorgiou, C., M. Saam, and P. Schulte. 2017. "Substitution between Clean and Dirty Energy Inputs: A Macroeconomic Perspective." *The Review of Economics and Statistics* 99 (2): 281–290.

Prywes, M. 1986. "A Nested CES Approach to Capital-energy Substitution." *Energy Economics* 8 (1): 22–28.

Temple, J. 2012. "The Calibration of CES Production Functions." *Journal of Macroeconomics* 34 (2): 294–303.

van der Werf, E. 2008. "Production Functions for Climate Policy Modeling: An Empirical Analysis." *Energy Economics* 30 (6): 2964–2979.